Optimal Ultra-Small Block-Codes for Binary Input Discrete Memoryless Channels

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Abstract

Optimal block-codes with a very small number of codewords are investigated for the binary input discrete memoryless channels. Those channels are the binary asymmetric channel (BAC), including the two special cases of the binary symmetric channel (BSC) and the Z-channel (ZC). The binary erasure channel (BEC) is a common used channel with ternary output. For the asymmetric channels, a general BAC, it is shown that so-called *flip codes* are optimal codes with two codewords. The optimal (in the sense of minimum average error probability, using maximum likelihood decoding) code structure is derived for the ZC in the cases of two, three, and four codewords and an arbitrary finite blocklength. For the symmetric channels, the BSC and the BEC, the optimal code structure is derived with at most three codewords and an arbitrary finite blocklength, a statement for linear optimal codes with four codes is also given.

The derivation of these optimal codes relies heavily on a new approach of constructing and analyzing the codebook matrix not row-wise (codewords), but *column-wise*. This new tool allows an elegant definition of interesting code families that is recursive in the blocklength n and admits their *exact* analysis of error performance that is not based on the union bound or other approximations.